Elburton Primary School

Progression in Calculation Policy

	February 2022	Policy Review Date
1	9th February 2020	Policy Agreed
Version	**Date**	**Description**

This Progression in Calculations Policy has been devised to support understanding in the expectations for fluency of the new National Curriculum in England (2014) as well as the progression of calculation concepts through a child's mathematical development.

Principles

- To support children's development and understanding of calculations through the use of concrete, pictorial and abstract methods.
- To support children to develop a deep understanding of number as well as mental and written calculation.
- To develop, using a Mastery Maths approach, number awareness and fluency, which is supported through the use of models and images.
- To ultimately develop proficiency with the expected formal written methods by the end of Year 6.
- Specific practical resources and models have been suggested for each age group to support children in developing the conceptual understanding that will enable them to move more rapidly and efficiently towards the formal written methods expected.
- It is expected that all children will work towards the fluency goals for each age group but that, where necessary, teachers will use approaches and materials from earlier year groups to bridge any gaps in a child's understanding.
- Teachers should have an understanding of the expectations and progression for all year groups, regardless of which year group they teach. All teachers have progression maps linked to the objectives set out in the National Curriculum 2014.

Concrete-Pictorial-Abstract Representations

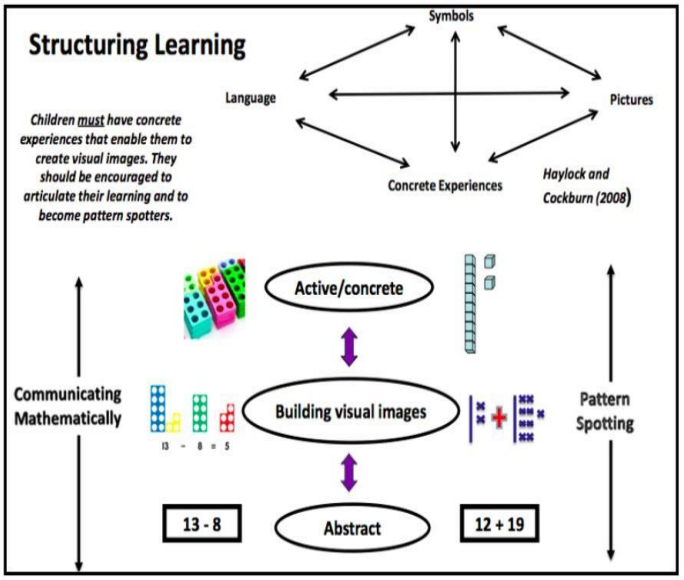
Children develop an understanding of a mathematical concept through the three steps (or representations) of concrete-pictorial-abstract approach.

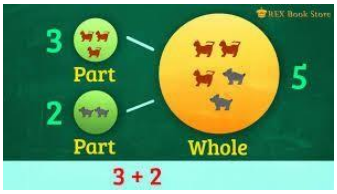
Concrete representation - a pupil is first introduced to an idea or a skill by acting it out with real objects.

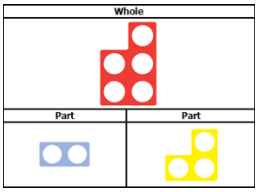
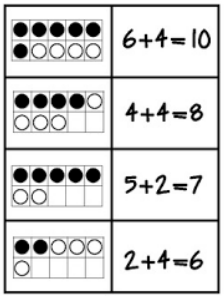
This is a 'hands on' component using real objects and it is the foundation for conceptual understanding.

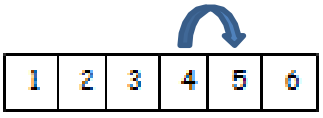
Pictorial representation - a pupil has sufficiently understood the hands-on experiences performed and can now relate them to representations, such as a diagram or picture of the problem.

Abstract representation - a pupil is now capable of representing problems by using mathematical notation, for example: 12 ÷ 2 = 6

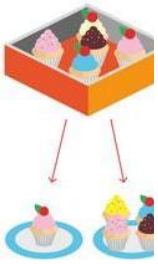
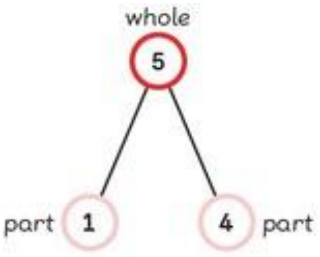
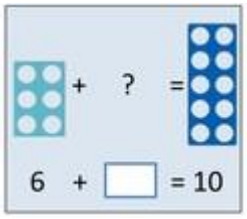
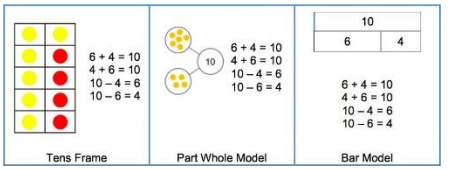
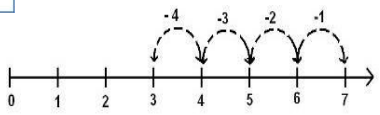
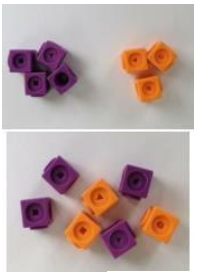
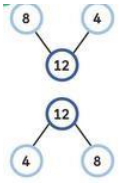
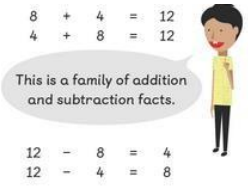


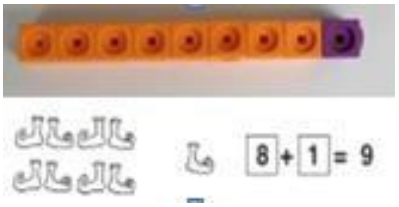
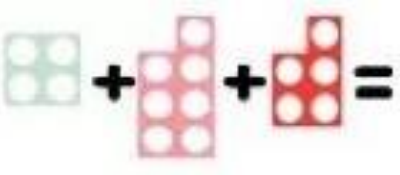
<table>
<tr><th colspan="2">Early Years Addition</th></tr>
<tr><td>Explore the part/part/whole relationship, using concrete materials.</td><td> 

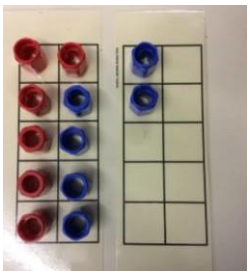

</td></tr>
<tr><td>Use ten frame to support their understanding of number and addition of single digits.</td><td> 
</td></tr>
<tr><td>Solve problems using concrete materials and pictorial images to support.</td><td> 

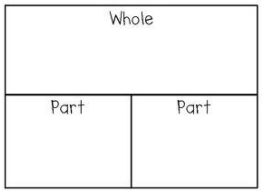
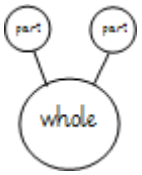
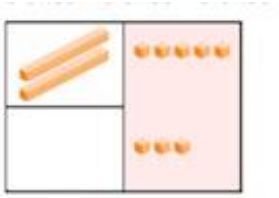
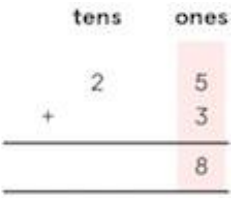
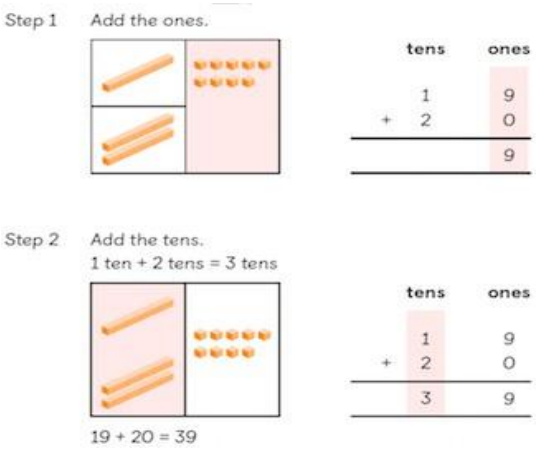
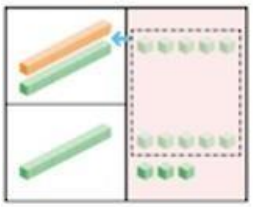
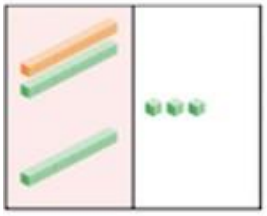
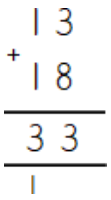

Which one has more?</td></tr>
<tr><td>Use number lines alongside number tracks and practical apparatus
to solve addition calculations and word problems.</td><td>What is 1 more than 4?
 

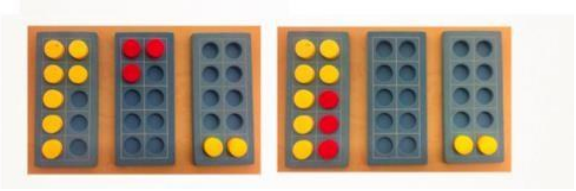
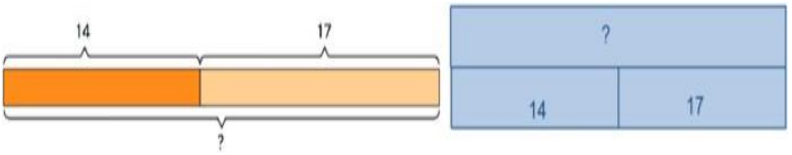

</td></tr>
<tr><td>Support children with reading number sentences aloud in different ways.</td><td>“Three add two equals five.” or “Five is <u>equal</u> to three and two.” or “Five is the <u>same as</u> three and two.”</td></tr>
<tr><td>Solve simple problems using fingers.</td><td> 
</td></tr>
<tr><td colspan="2"><u>Key Vocabulary</u>
Estimate; add; more; and; total; make; altogether; part-part-whole;
double; one more, two more; how many more make _____?; same as; how many more is __ than _____?</td></tr>
</table>

Year 1 Addition	
Understand the equals sign and the concept of equality before using the '=' sign.	***2 = 1 + 1*** ***2 + 3 = 4 + 1*** ***Written calculations should be written either side of the equality sign so that the sign is not just interpreted as 'the answer'.***
Join two groups and then recount all objects using one-to-one correspondence. Use concrete resources alongside part/part/whole model and number lines to support number sentences.	  *5 = 4 + 1* *3 + 4 = 7*
Use concrete apparatus to learn the number bonds within 10.	
Use ten frames, part/ part/whole and bar model to learn number bonds within 20 and demonstrate related facts. *Teach addition and subtraction alongside each other as pupils need to see the relationship between the facts.*	    

<table>
<tr><td>Use a range of concrete resources (including multi-link and Numicon) to support adding one or two digit numbers to 20, including zero.</td><td>   </td></tr>
<tr><td>Use ten frames to support bridging 10.

Children should start with the larger number and add the smaller number, seeing what makes ten and what is left over.</td><td> 
6 + 6 = 12
6 + 4 = 10
10 + 2 = 12</td></tr>
<tr><td colspan="2"><u>Formal Written Expectation:</u>
Read, write and interpret mathematical statements involving addition (+), subtraction (–) and equals (=) signs.
Add and subtract one-digit and two-digit numbers to 20, including zero.
Represent and use number bonds and related subtraction facts within 20.</td></tr>
<tr><td colspan="2"><u>Key Vocabulary:</u>
Add; plus; more; more than; altogether; total; sum; and; part-part-whole; put together</td></tr>
</table>

Year 2 Addition	
Use concrete objects and pictorial representations to add a 2 digit number with a 1 digit number.	   
Use concrete objects and pictorial representations to add two 2 digit numbers (with no renaming required).	 ***Replace Dienes with place value counters as well.***
Use concrete objects and pictorial representations to add two 2 digit numbers, with renaming required.	Step 1: Add the ones. 5 ones + 8 ones = 13 ones Regroup the ones. 13 ones = 1 ten and 3 ones  Step 2: Add the tens. 1 ten + 1 ten + 1 ten = 3 tens  ***Once this concept has been explored with Dienes and place value counters physically, move onto the formal written column method. Some children may initially need to continue to use the Dienes alongside the written method for support.*** 

Use concrete objects and pictorial representations to add three single digit numbers.	7+3+2 = Leads to 10 + 2 = 
Use the bar model to find missing quantities.	Helen has 14 bread-sticks. Her friend has 17. How many do they have altogether? 

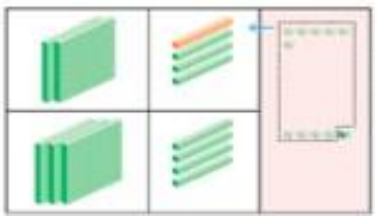
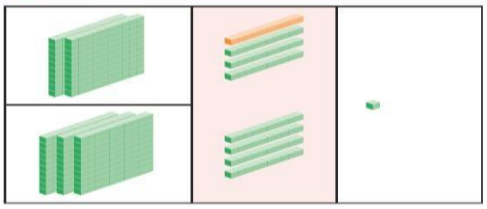
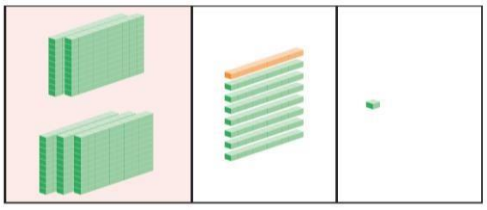
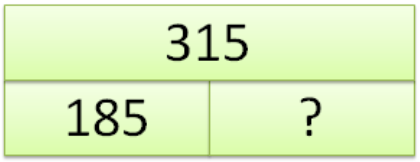
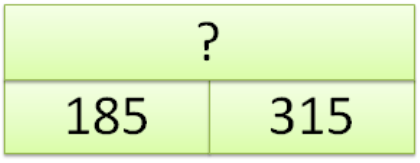
Formal Written Expectation:

Add and subtract numbers using concrete objects, pictorial representations, and mentally, including:

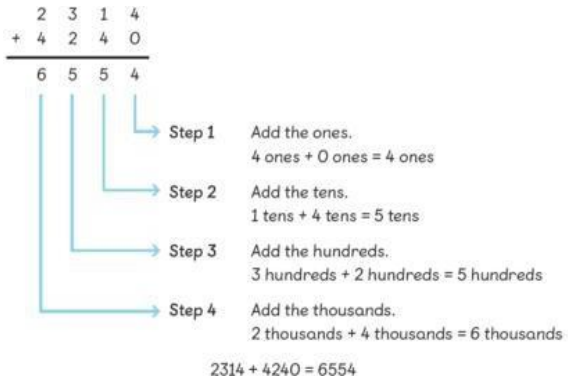
- ***a two-digit number and ones***
- ***a two-digit number and tens***
- ***two two-digit numbers***
- ***adding three one-digit numbers***

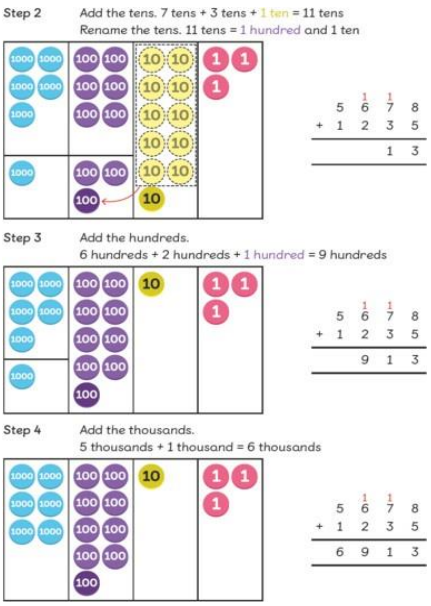
Key Vocabulary:

Add; plus; more; more than; altogether; total; sum; and; part-part-whole; regroup; tens; ones

Year 3 Addition	
Add two three digit numbers. *Children need to use equipment first to support their understanding of place value.* *Children move to three digits + three digits, starting without renaming, and moving towards renaming as appropriate.*	236 + 345 =  H T O 2 3 6 + 5 4 5 1 1 Step 2 Add the tens. 1 ten + 3 tens + 4 tens = 8 tens  H T O 2 3 6 + 3 4 5 8 1 1 Step 3 Add the hundreds. 2 hundreds + 3 hundreds = 5 hundreds  H T O 2 3 6 + 3 4 5 5 8 1 1 236 + 345 = 581
Use the bar to find missing digits.	 315 – 185 = ? 185 + ? = 315  185 + 315 = ? ? – 185 = 315
Formal Written Expectation: ***Add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction***	
Key Vocabulary: ***Add; addition; more; plus; make; sum; total; altogether; score; double; near double; one more, two more... ten more... one hundred more; how many more to make…?; regroup; hundreds; tens; ones***	

<table>
<tr><th colspan="2">Year 4 Addition</th></tr>
<tr><td>Add numbers with up to 4 digits.

It is important to use place value counters (and dienes if still necessary) to support understanding of the structure.</td><td>




</td></tr>
<tr><td>Use the bar to find missing digits.

It is important for children to use the bar model in this way to encourage the use of it with problem solving.</td><td>This is not a form of getting the correct answer but helping to guide children to the correct operation.

Alison jogs 6,860 metres and Calvin jogs 5,470 metres. How far do they jog altogether?

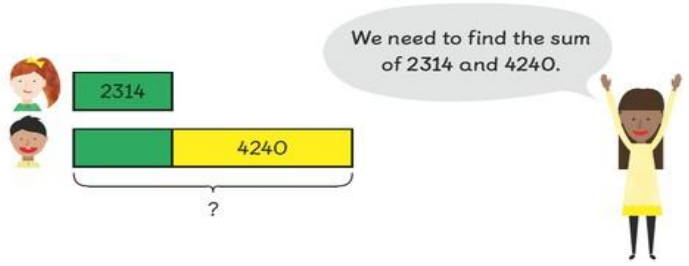
 
</td></tr>
<tr><td colspan="2"><u>Formal Written Expectation:</u>

Add and subtract numbers with up to 4 digits using the formal written methods of columnar addition where appropriate</td></tr>
<tr><td colspan="2"><u>Key Vocabulary:</u>

Add; addition; more; plus; increase; sum; total; altogether; score; double; near double; regroup; thousand; hundreds; tens; ones</td></tr>
</table>

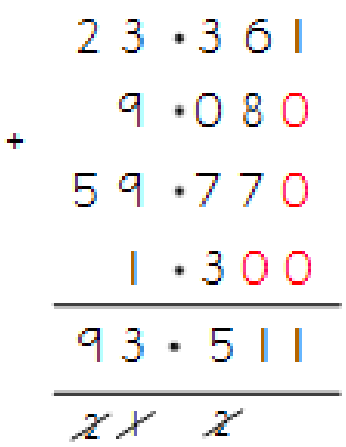
<table>
<tr><th colspan="2">Year 5 Addition</th></tr>
<tr><td>Add numbers with more than 4 digits including decimals.

It is important to use place value charts and counters to support understanding of the structure.</td><td>Example 1:
<table>
<tr><td></td><td>2</td><td>3</td><td>.</td><td>5</td><td>9</td></tr>
<tr><td>+</td><td></td><td></td><td>7</td><td>.</td><td>5</td><td>5</td></tr>
<tr><td></td><td>3</td><td>1</td><td>.</td><td>1</td><td>4</td></tr>
<tr><td></td><td></td><td>1</td><td>1</td><td></td><td>1</td></tr>
</table>
Example 2:
<table>
<tr><td></td><td>2</td><td>3</td><td>4</td><td>8</td><td>1</td></tr>
<tr><td>+</td><td></td><td></td><td>1</td><td>3</td><td>6</td><td>2</td></tr>
<tr><td></td><td>2</td><td>4</td><td>8</td><td>4</td><td>3</td></tr>
<tr><td></td><td></td><td></td><td></td><td>1</td><td></td></tr>
</table>
Example 3:
<table>
<tr><td></td><td>1</td><td>9</td><td>.</td><td>0</td><td>1</td></tr>
<tr><td>+</td><td></td><td>3</td><td>.</td><td>6</td><td>5</td></tr>
<tr><td></td><td></td><td>0</td><td>.</td><td>7</td><td>0</td></tr>
<tr><td></td><td>2</td><td>3</td><td>.</td><td>3</td><td>6</td></tr>
<tr><td></td><td></td><td>1</td><td>1</td><td></td><td></td></tr>
</table></td></tr>
<tr><td>Use the bar to help find missing digits.

It is important for children to use the bar model in this way to encourage the use of it to aid with problem solving.</td><td> 
</td></tr>
<tr><td colspan="2"><u>Formal Written Expectation:</u>

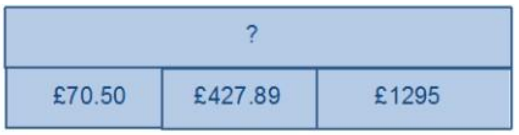
Add and subtract whole numbers with more than 4 digits, including using formal written methods (columnar addition and subtraction)</td></tr>
<tr><td colspan="2"><u>Key Vocabulary:</u>

Add; addition; more; plus; increase; sum; total; altogether; score; double; near double; how many more to make…?; regroup; thousands; hundreds; tens; ones</td></tr>
</table>

<table>
<tr><th colspan="2">Year 6 Addition</th></tr>
<tr><td>Add several numbers with up to three decimal places.</td><td>


Empty decimal places should be filled with zeros as place value holders.

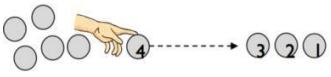
When adding several numbers which have different decimal places (including money and measures), tenths, hundredths and thousandths should be correctly aligned, with the decimal point lined up vertically (including in the answer row).
</td></tr>
<tr><td>Add using the bar.</td><td>
Jack went on holiday. His flight cost £70.50, the hotel £1295 and spending money £427.89. How much did Jack spend on his holiday?

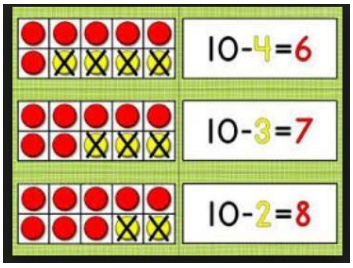


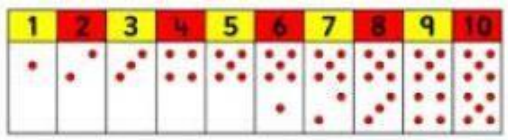
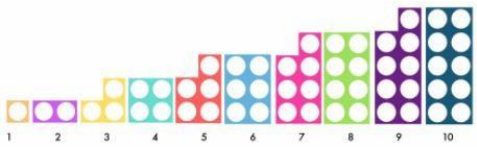
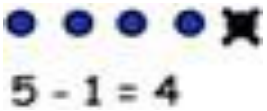
</td></tr>
<tr><td colspan="2"><u>Formal Written Expectation:</u>

Solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why</td></tr>
<tr><td colspan="2"><u>Key Vocabulary:</u>

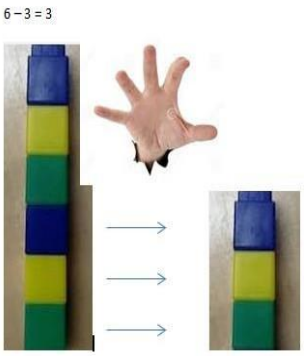
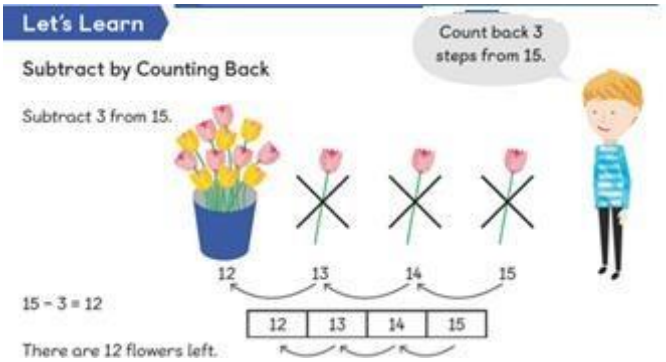
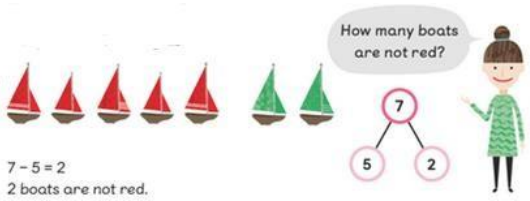
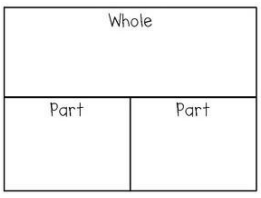
Add; addition; more; plus; increase; sum; total; altogether; score; double; near double; how many more to make…?; regroup; thousands; tens; hundreds; ones</td></tr>
</table>

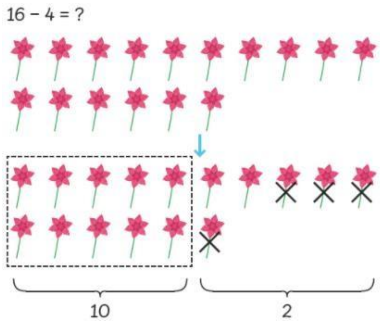
<table>
<tr><th colspan="2">Early Years Subtraction</th></tr>
<tr><td>Use concrete resources to help build and reinforce the concept of taking away.</td><td>Taking away after counting out practical equipment. . Children would be encouraged to physically remove these using touch counting.
 
By touch counting and dragging in this way, it allows children to keep track of how many they are removing so they don't have to keep recounting. They will then touch count the amount that are left to find the answer.

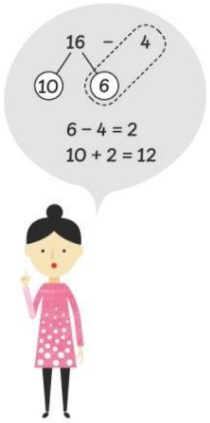
Those who are ready may record their own calculations</td></tr>
<tr><td>Use a ten frame to support subtraction by taking away:</td><td>  </td></tr>
<tr><td>Introduce number tracks and number lines alongside concrete resources to count back and to find one less.

Children show hops on a number line for finding more than one less.</td><td>What is 1 less than 9?
 
 
What is 3 less than 10?
  </td></tr>
<tr><td>Construct pictures and number sentences to go with practical activities</td><td> 
  </td></tr>
<tr><td>Solve simple problems using fingers.</td><td>  </td></tr>
<tr><td colspan="2"><u>Key Vocabulary:</u>

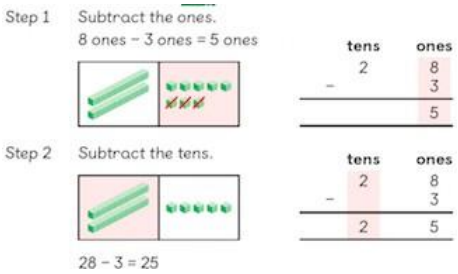
Estimate; take (away); leave; how many are left / left over?; how many have gone?; part-part-whole; one less, two less?; how many fewer is … than…?; the same as; counting/hopping back</td></tr>
</table>

Year 1 Subtraction	
Understand the equals sign and the concept of equality before using the '=' sign.	***4 = 5 - 1*** ***6 – 2 = 8 - 4***  ***Written calculations should be written either side of the equality sign so that the sign is not just interpreted as 'the answer'.***
Use concrete resources and models alongside each other to support understanding of difference.	 
Subtract a single digit number from a single digit number and a single digit from a two digit by crossing out pictures.	 ***7 – 2 = 5*** ***There are now 5 ladybirds left.***
Subtract by counting back on a number line or track.	
Use the part/part/whole and bar model to support interpretation of the calculation required in problems.	 

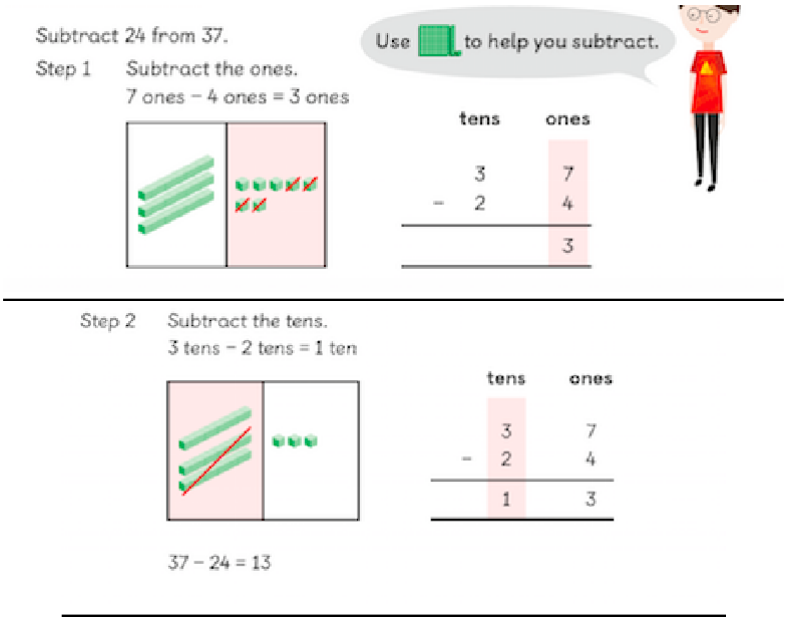
<table>
<tr>
<td>Subtract ones from a number greater than ten by subtracting from the ones.</td>
<td> 

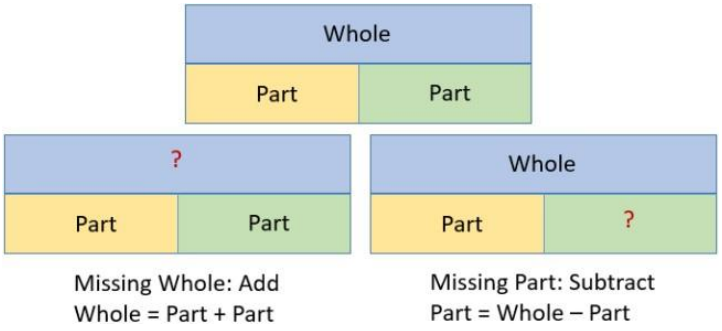

</td>
</tr>
<tr>
<td>When subtracting using Dienes or place value counters, children need to be taught to rename a 1 ten for 10 ones and then subtract from those ones.</td>
<td> 

20 – 4 = 16</td>
</tr>
<tr>
<td colspan="2"><u>Formal Written Expectation:</u>
Read, write and interpret mathematical statements involving addition (+), subtraction (–) and equals (=) signs.
Add and subtract one-digit and two-digit numbers to 20, including zero.
Represent and use number bonds and related subtraction facts within 20.</td>
</tr>
<tr>
<td colspan="2"><u>Key Vocabulary</u>
Subtract; minus; less than; fewer; take away; how many are left?; how many have gone?; difference between; distance between</td>
</tr>
</table>

<table>
<tr><th colspan="2">Year 2 Subtraction</th></tr>
<tr><td>Use concrete objects and pictorial representations to subtract a 1 digit number from 2 digit number.</td><td>


</td></tr>
<tr><td>Use concrete objects and pictorial representations to subtract a 2-digit number from 2 digit number.

Including examples which require renaming.</td><td>


</td></tr>
<tr><td>Use the bar model to help recognise and use the inverse relationship between addition and subtraction.</td><td>


</td></tr>
<tr><td colspan="2"><u>Formal Written Expectation:</u>

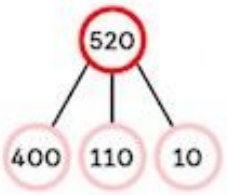
Add and subtract two two-digit numbers using concrete objects, pictorial representations progressing to formal written methods</td></tr>
<tr><td colspan="2"><u>Key Vocabulary:</u>

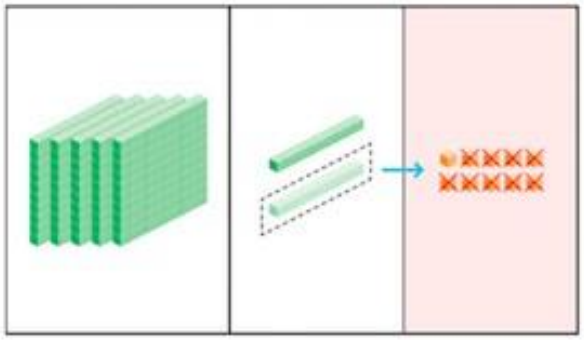
Subtract; minus; less than; fewer; take away; how many are left?; how many have gone?; difference between; distance between; regroup; tens; ones</td></tr>
</table>

<table>
<tr><th colspan="2">Year 3 Subtraction</th></tr>
<tr><td>Subtract up to 3 digits from 3 digits.

Children move to three digits - three digits, starting without renaming, and moving towards renaming as appropriate.</td><td>Subtract 269 from 520.

Step 1 Regroup 1 ten into 10 ones.
Subtract the ones.
10 ones − 9 ones = 1 one

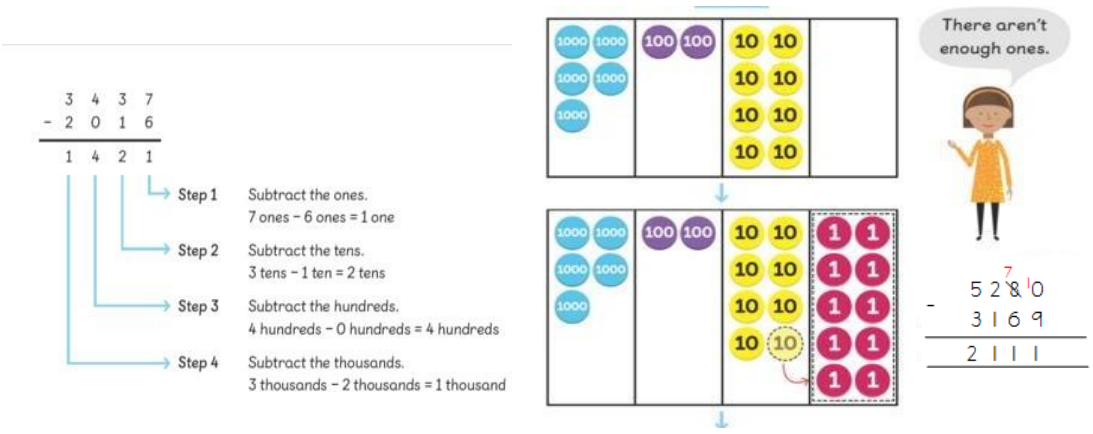
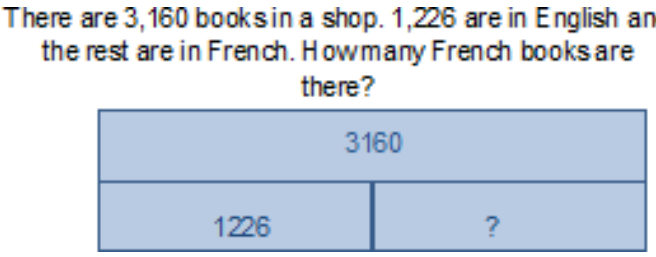
<table>
<tr><th></th><th>h</th><th>t</th><th>o</th></tr>
<tr><td></td><td>5</td><td>1 2</td><td>10 0</td></tr>
<tr><td>−</td><td>2</td><td>6</td><td>9</td></tr>
<tr><td></td><td></td><td></td><td>1</td></tr>
</table></td></tr>
<tr><td>Use the bar to find missing digits.

It is important for children to use the bar model in this way to encourage the use of it with problem solving.</td><td>
<table>
<tr><td colspan="2">315</td></tr>
<tr><td>185</td><td>?</td></tr>
</table>
315 – 185 = ?
185 + ? = 315

<table>
<tr><td colspan="2">?</td></tr>
<tr><td>185</td><td>315</td></tr>
</table>
185 + 315 = ?
? – 185 = 315
</td></tr>
<tr><td colspan="2"><u>Formal Written Expectation:</u>

Add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction</td></tr>
<tr><td colspan="2"><u>Key Vocabulary:</u>

How many more is… than…? how much more is…? –, subtract, subtraction, take (away), minus, leave, how many are left/left over? one less, two less… ten less… one hundred less, how many fewer is… than…? how much less is…? difference between, half, halve, equals, the same as; regroup; hundreds; tens; ones</td></tr>
</table>

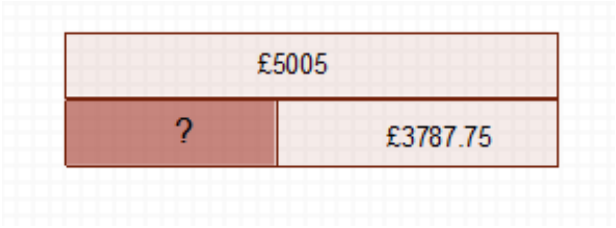
Year 4 Subtraction	
To subtract with numbers up to four digits, including exchanging when children are secure. *It is important to use place value counters (and dienes if still necessary) to support understanding of the structure.*	
Use the bar to find missing digits. *It is important for children to use the bar model in this way to encourage the use of it with problem solving.*	
Formal Written Expectation: ***Add and subtract numbers with up to four digits using the formal written methods of columnar addition where appropriate***	
Key Vocabulary: ***How many more to make…? subtract, subtraction, take (away), minus, decrease, leave, how many are left/left over? difference between, half, halve, how many more/fewer is… than…? how much more/less is…? equals, the same as, inverse; regroup; thousands; hundreds; tens; ones***	

<table>
<tr><th colspan="2">Year 5 Subtraction</th></tr>
<tr><td>Subtract with at least four digit numbers including two decimal places.

Include examples involving money, measures and decimals -ensuring that children do this practically as well as in the abstract.</td><td>Subtract with decimal values, including mixtures of integers and decimals, aligning the decimal point.

2 10 4
3 X 10 5 16
- 2 1 2 8
2 8 9 2 8

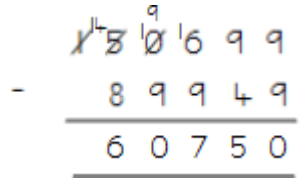
6 10 8
7 X 16 9 • 10
- 3 7 2 • 5
6 7 9 6 • 5</td></tr>
<tr><td>Use the bar to find missing digits.</td><td>A holiday to Spain costs £5005 for a family of four. The Smiths have only saved £3787.75. How much money do they still need?

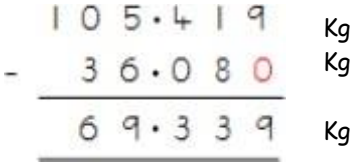
 
</td></tr>
<tr><td colspan="2"><u>Formal Written Expectation:</u>

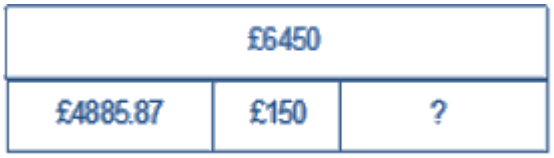
Add and subtract whole numbers with more than 4 digits, including using formal written methods (columnar addition and subtraction)</td></tr>
<tr><td colspan="2"><u>Key Vocabulary:</u>

Subtract; subtraction; take (away); minus; decrease; leave; how many are left/left over? difference between; half; halve; how many more/fewer is… than…? how much more/less is…?; regroup; thousands; hundreds; tens; ones</td></tr>
</table>

<table>
<tr><th colspan="2">Year 6 Subtraction</th></tr>
<tr><td>Subtract with increasingly large, more complex numbers and decimal values.</td><td>Very important to use in a range of contexts- measures and money.

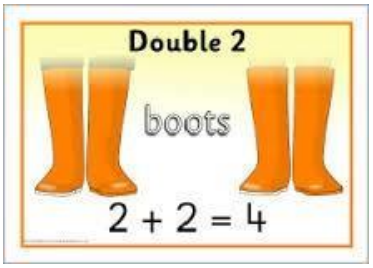
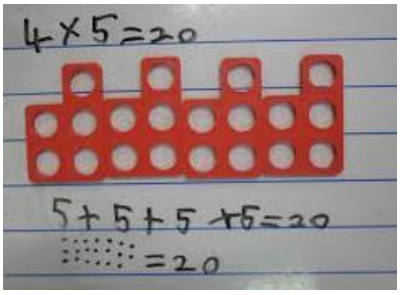
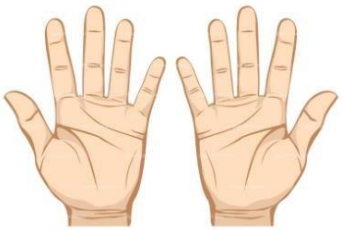
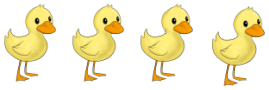



</td></tr>
<tr><td>Use the bar model for subtraction.</td><td>Chloe wants to buy a new car for £6450. She has £4885.87 in her savings account. Her Dad gives her £150 for her birthday. How much more money does she need to save?


</td></tr>
<tr><td colspan="2"><u>Formal Written Expectation:</u>

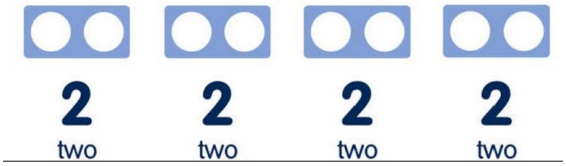
Solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why</td></tr>
<tr><td colspan="2"><u>Key Vocabulary:</u>

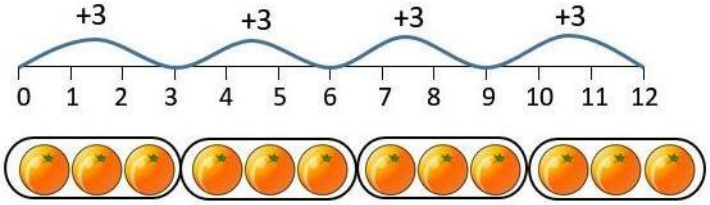
Subtract; subtraction; take (away); minus; decrease; leave; how many are left/left over? difference between; half; halve; how many more/fewer is… than…? how much more/less is…? Equals; is the same as; inverse; regroup; thousands; hundreds; tens; ones</td></tr>
</table>

<table>
<tr><th colspan="2">Early Years Multiplication</th></tr>
<tr><td>Introduce the link between addition and multiplication through doubling.</td><td> 
</td></tr>
<tr><td>Use Numicon to visualise the repeated adding of the same number. (These might be drawn around or printed as a way of recording.)</td><td> 
</td></tr>
<tr><td>Use real life contexts and use of practical equipment
to count in repeated groups of the same size.</td><td>How many fingers on two hands? How many sides on three triangles? How many legs on four ducks?
 
 </td></tr>
<tr><td colspan="2"><u>Key Vocabulary:</u>
Lots of; groups of; once, twice, three times… ten times…as (big, long, wide… and so on);
double; estimate; add again and again; part-part-whole</td></tr>
</table>

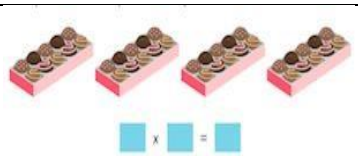
<table>
<tr><th colspan="2">Year 1 Multiplication</th></tr>
<tr><td>Count in multiples of 2, 5 and 10 from zero, using concrete resources and diagrams.

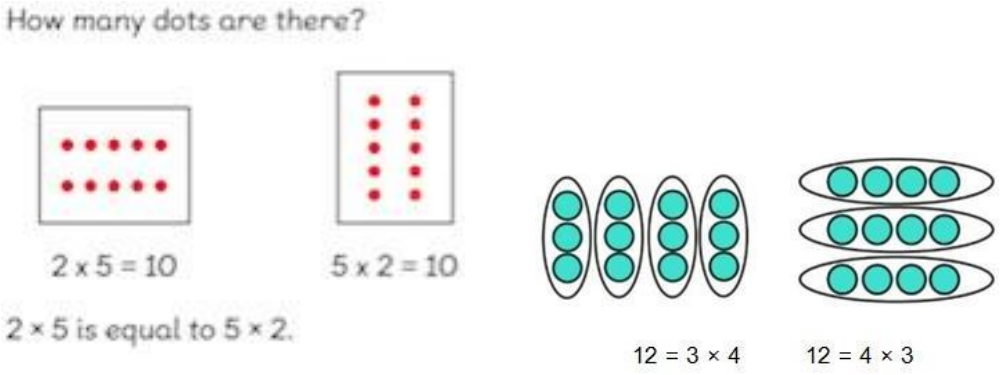
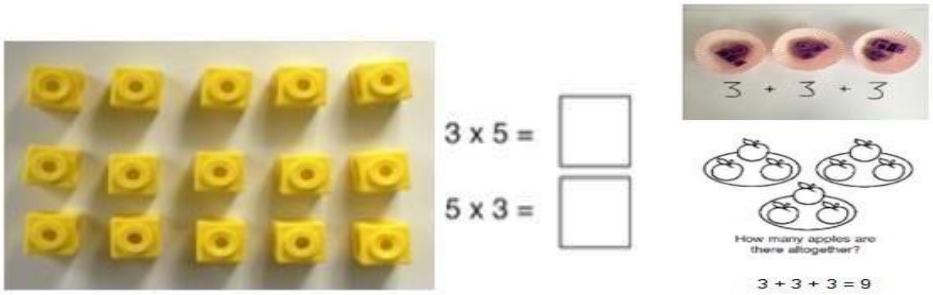
Extend to presenting and recording using arrays.

Children could count the number of groups on their fingers as they are skip counting.</td><td> 
 

 
 
2 × 4 = 8</td></tr>
<tr><td>Solving multiplication problems using repeated addition, and use a number line to support understanding.</td><td>How many legs do these dogs have altogether?
 
4 + 4 + 4 which makes 12!
3 x 4 is the same as 3 + 3 + 3 + 3
 
</td></tr>
<tr><td colspan="2"><u>Formal Written Expectation:</u>
N/A</td></tr>
<tr><td colspan="2"><u>Key Vocabulary:</u>
Lots of; groups of; times; multiply; multiplied by;

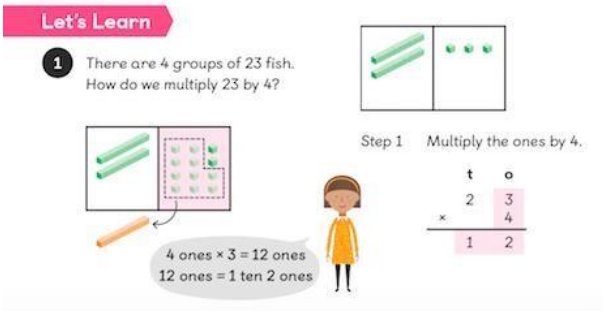
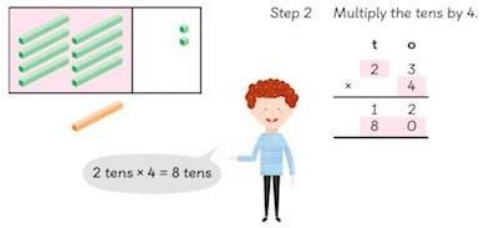
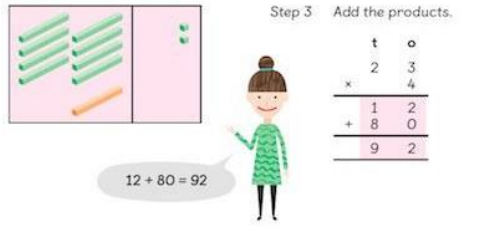
once, twice, three times…as (big, long, wide… and so on); double; add again and again</td></tr>
</table>

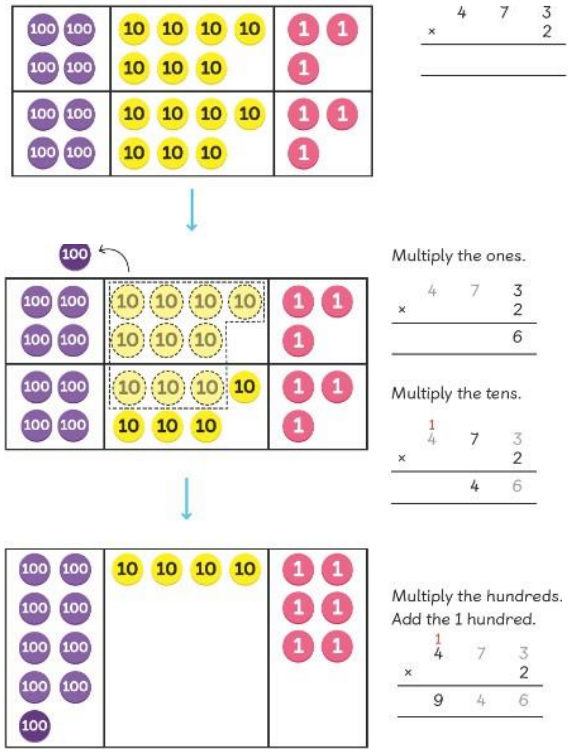
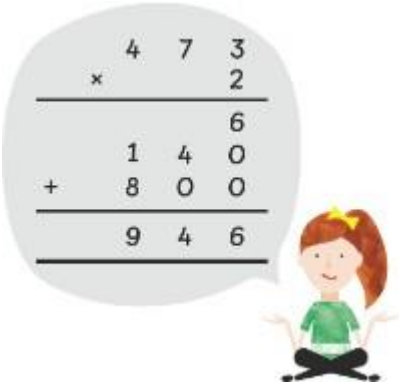
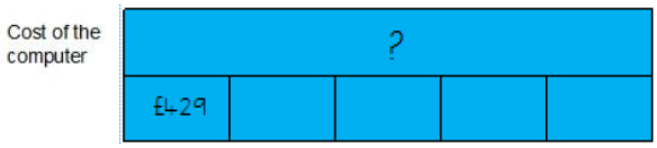
<table>
<tr><th colspan="2">Year 2 Multiplication</th></tr>
<tr><td>Skip counting in multiples of 2, 3, 5, 10 from 0.</td><td> 
</td></tr>
<tr><td>Recall and use multiplication facts for the multiplication tables 2, 5 and 10.</td><td> 
</td></tr>
<tr><td>Use multiplication (x) and equal (=) sign when recording calculations.</td><td>  </td></tr>
<tr><td>Understand that multiplication is commutative.

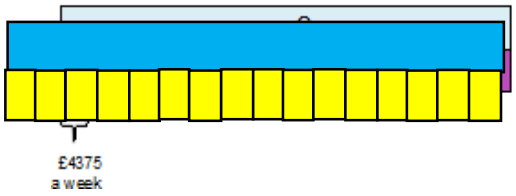
Pupils should understand that an array can represent different equations and that, as multiplication is commutative, the order of the multiplication does not affect the answer.</td><td> 
</td></tr>
<tr><td>Solve multiplication problems in context, using arrays and repeated addition.</td><td> 
</td></tr>
<tr><td colspan="2"><u>Formal Written Expectation:</u>

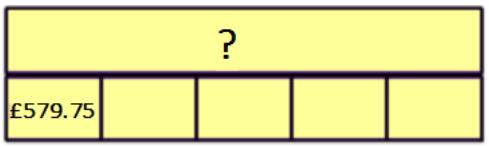
Calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication (×), division (÷) and equals (=) signs</td></tr>
<tr><td colspan="2"><u>Key Vocabulary:</u>

Lots of; groups of; times; multiply; multiplied by; once, twice, three times…as (big, long, wide… and so on); double; add again and again</td></tr>
</table>

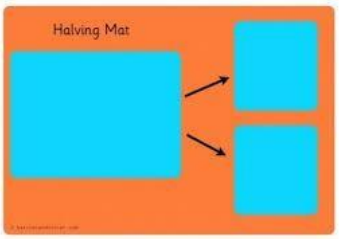
Year 3 Multiplication	
Multiply a two digit number by a one digit, using the expanded column method initially. Move on to the short column method when secure with understanding. *Children should be able to recall the 2, 5, 10, 3, 4 and 8 times tables.*	 *Example short column method:* 4 3 x 5 2 1 5 1   23 × 4 = 92 There are 92 fish in 4 tanks.
Use the bar to solve multiplication problems.	4 children go to the cinema. They each pay £15. How much do they spend altogether? Whole unknown 
Formal Written Expectation: ***Write and calculate mathematical statements for ÷ using the x tables they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods***	
Key Vocabulary: ***Lots of; groups of ×; times; multiply; multiplication; multiplied by; multiple of; product; once, twice, three times… ten times… times as (big, long, wide… and so on); repeated addition; array; row; column; double***	

Year 4 Multiplication	
Multiply a three digit number by a one digit, using the expanded column method initially (and support the understanding of this though using place value counters). Move on to the short column method when secure with understanding. *Children to know all times tables to 12 x 12.*	  1 4 3 x 5 7 1 5 2 1
Use the bar to aid the solving of multiplication problems. *It is important for children to use the bar model in this way to encourage the use of it with problem solving.*	A computer costs 5 times as much as a television. The television costs £429. How much does the computer cost? 
Formal Written Expectation: ***Multiply two-digit and three-digit numbers by a one-digit number using formal written layout***	
Key Vocabulary: ***lots of, groups of, times, multiply, multiplication, multiplied by, multiple of, product, once, twice, three times… ten times… times as (big, long, wide… and so on) repeated addition, array, row, column, double; regroup; thousands; hundreds; tens; ones***	

Year 5 Multiplication	
Multiply up to four digit numbers by two digits using long multiplication. *Children need to be taught to estimate first, (e.g. for 72 x 38 they will use rounding: 72 x 38 is approximately 70 x 40 = 2800) and use their estimation to check the reasonableness of their answer.*	56 x 27 392 (56 x 7) 1120 (56 x 20) 1512 (56 x 27) · Explain that first we are multiplying the top number by 7 starting with the ones. (Any carrying needs to be done underneath the numbers.) · Now explain that we need to "drop the zero"—explain that this is because we are multiplying the number by 20 (which is 2 tens) and this is the same as multiplying by 10 and then 2. · Now add the 2 numbers together to give you the answer. This will need lots of modeling for children to fully understand what is actually happening, and that it's not a trick! 3652 x 8 29216 1234 x 16 7404 (1234 x 6) +12340 (1234 x 10) 19744 (1234 x 16)
Use the bar to support multiplication . *It is important for children to use the bar model in this way to encourage the use of it to aid with problem solving.*	The cost to run a sports centre is £4375 a week, how much would it cost to run for 16 weeks? 
Formal Written Expectation: ***Multiply numbers up to four digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers***	
Key Vocabulary: ***Lots of; groups of; times; multiply; multiplication; multiplied by; multiple of; product; once, twice, three times… ten times… times as (big, long, wide… and so on); repeated addition; array; row; column; double***	

<table>
<tr><th colspan="2">Year 6 Multiplication</th></tr>
<tr><td>Short and long multiplication with up to two decimal places.</td><td>
<pre>
  56
x 27
 392   (56 x 7)
1120   (56 x 20)
1512   (56 x 27)
 1
</pre>
<pre>
  3·19
x   8
25·52
 1 2
</pre>
</td></tr>
<tr><td>Use the bar model to help with multiplication.</td><td>If 5 friends went on holiday and each paid £579.75, what was the total cost of the holiday?
 
</td></tr>
<tr><td colspan="2"><u>Formal Written Expectation:</u>
Multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication</td></tr>
<tr><td colspan="2"><u>Key Vocabulary:</u>
Lots of; groups of; times; multiply; multiplication; multiplied by; multiple of; product; once, twice, three times… ten times… times as (big, long, wide… and so on); repeated addition; array; row</td></tr>
</table>

<table>
<tr><th colspan="2">Early Years Division</th></tr>
<tr><td>Introduce division through halving;
use practical resources and models to support this.</td><td> 



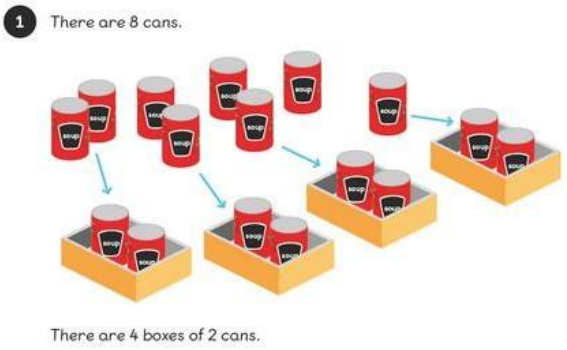
Physically share objects between children.</td></tr>
<tr><td>Begin with mostly pictorial representations linked to real life contexts.</td><td> 
 </td></tr>
<tr><td>Children need to see and hear representations of halving as <u>sharing</u>.

(Setting the problems in a real life context and solving them with concrete apparatus will support children's understanding.)</td><td><u>Sharing:</u>
I have 10 sweets. I want to share them with my friend. How many will we have each?
  </td></tr>
<tr><td colspan="2"><u>Key Vocabulary:</u>

Halve; share; share equally; one each, two each, three each…; part-part-whole;

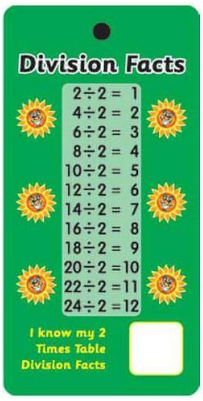
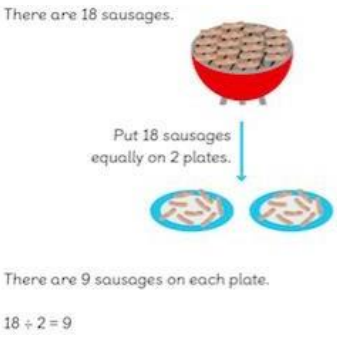
equal groups of; left; left over; estimate; half; halves; whole</td></tr>
</table>

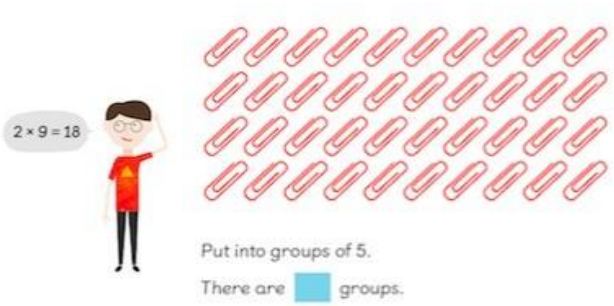
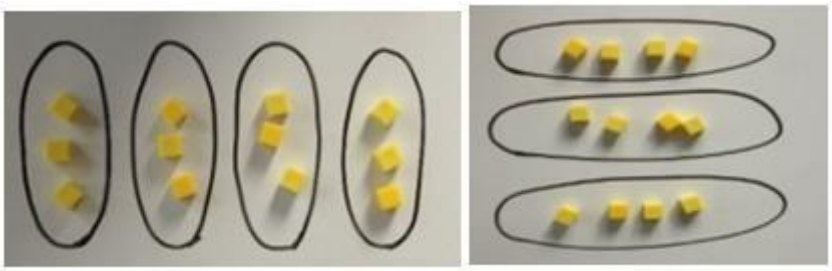
<table>
<tr><th colspan="2">Year 1 Division</th></tr>
<tr><td>Pupils should be taught to divide through working practically and the sharing should be shown below the whole to familiarize children with the concept of the whole.

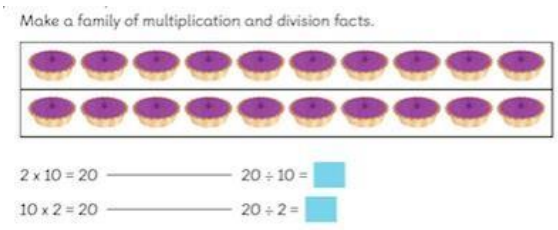
The language of whole and part, part should be used.</td><td>8 ÷ 4 = 2
 
</td></tr>
<tr><td>Children need to see and hear representations of division as both <u>grouping</u> and <u>sharing</u>.

(Setting the problems in a real life context and solving them with concrete apparatus will support children's understanding.)</td><td><u>Grouping model:</u>
Mum has 6 socks. She grouped them into pairs. How many pairs did she make?
How many socks did she have altogether?
 

<u>Sharing model:</u>
I have 10 sweets. I want to share them with my friend. How many will we have each?
  </td></tr>
<tr><td colspan="2"><u>Formal Written Expectation:</u>
N/A</td></tr>
<tr><td colspan="2"><u>Key Vocabulary</u>
Count in twos, threes, fives, tens (forwards from/backwards from); how many times? lots of;
groups of; once, twice, three times, five times; times; multiply; multiply by; repeated addition</td></tr>
</table>

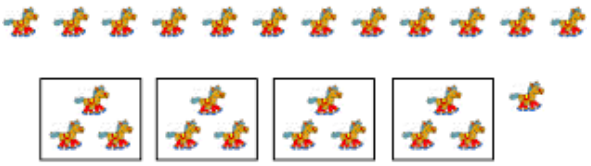
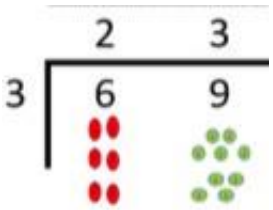
<table>
<tr><th colspan="2">Year 2 Division</th></tr>
<tr><td>Recall and use division facts for the multiplication tables 2, 5 and 10.</td><td> 
</td></tr>
<tr><td>Solve division problems in context using concrete objects by both sharing and grouping.</td><td>Sharing:
 

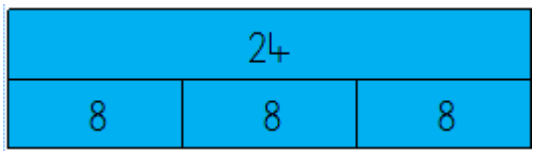
Grouping:
 
</td></tr>
<tr><td>Solve division problems in context using arrays.</td><td>  </td></tr>
<tr><td>Use the inverse.

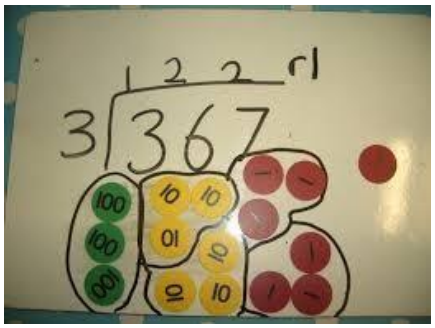
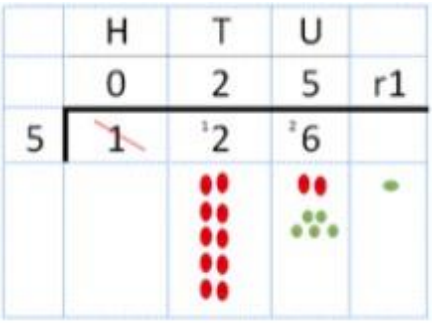
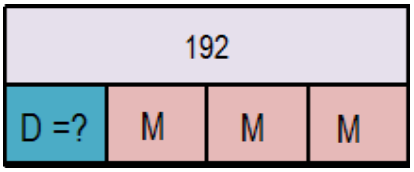
This should be taught alongside both multiplication and division.</td><td> 
</td></tr>
<tr><td colspan="2"><u>Formal Written Expectation:</u>

Calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication (×), division (÷) and equals (=) signs</td></tr>
<tr><td colspan="2"><u>Key Vocabulary:</u>

Count in twos, threes, fives, tens (forwards from/backwards from) how many times? lots of; groups of; once, twice, three times, five times; times; multiply; multiply by; repeated addition</td></tr>
</table>

<table>
<tr><th colspan="2">Year 3 Division</th></tr>
<tr><td>Divide by grouping and understand the concept of remainders.</td><td>Start with using the real objects-or objects that represent the calculation:
 
13 ÷ 4 = 3 Remainder 1</td></tr>
<tr><td>Divide using short division.
Once children are secure with division as grouping and can demonstrate this using number lines, arrays etc., short division for larger 2- digit numbers should be introduced. Initially, with carefully selected examples requiring no calculating of remainders at all. Start by introducing the layout of short division by including concrete apparatus (see diagram).</td><td> 
Remind children of correct place value, that 69 is equal to 60 and 9, but in short division, pose:
· We have six tens, how many groups of three can we make? 2
· We have nine ones, how many groups of three can we make? 3</td></tr>
<tr><td>Use the bar model to aid solving division problems.

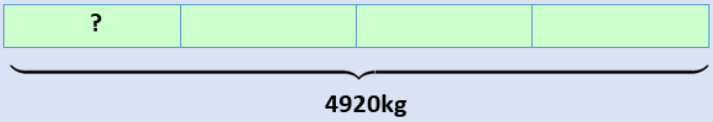
It is important for children to use the bar model in this way to encourage the use of it with problem solving.</td><td>Mrs Cook shared 24 slices of pizza between 3 children. How many slices did each child have?
  </td></tr>
<tr><td colspan="2"><u>Formal Written Expectation:</u>
Write and calculate mathematical statements for ÷ using the x tables they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods</td></tr>
<tr><td colspan="2"><u>Key Vocabulary:</u>
Halve; share; share equally; one each, two each, three each…; group in pairs, threes… tens; equal groups of; divide; division; divided by; divided into; left; left over; remainder</td></tr>
</table>

Year 4 Division	
Divide up to three digit numbers by a one digit number using short division. *Only when the children are secure with dividing a two digit number should they move onto a three digit number.* *It is important to use place value counters to support understanding of the structure.*	 
Use the bar to aid the solving of division problems.	 Desmond and Melissa collect cars. They have 192 cards in all. Melissa has three times as many cards as Desmon. How many cards does Desmondhave?
Formal Written Expectation: ***Calculate mathematical statements for division using the multiplication tables that they know, using mental and progressing to formal written methods***	
Key Vocabulary: ***Halve; share; share equally; one each, two each, three each…; group in pairs, threes… tens; equal groups of; divide; division; divided by; divided into; remainder; factor; quotient; divisible by; inverse***	

<table>
<tr><th colspan="2">Year 5 Division</th></tr>
<tr><td>Divide with up to four digit numbers by one digit including numbers where remainders are left.</td><td>0663 r 5
8 ⟌ 5 3 5 0 2 9

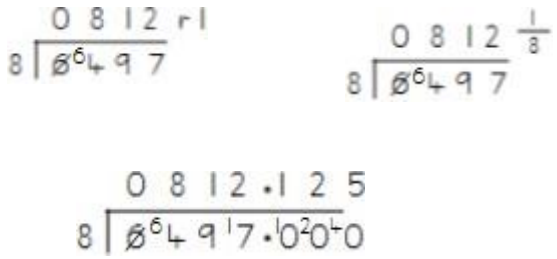
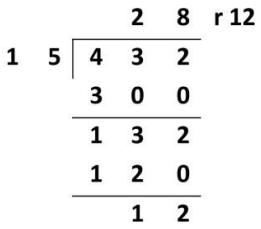
Short division with remainders: Now that pupils are introduced to examples that give rise to remainder answers, division needs to have a real life problem solving context, where pupils consider the meaning of the remainder and <u>how</u> to express it (i.e. as a fraction, a decimal, or as a rounded number or value: depending upon the context of the problem).</td></tr>
<tr><td>Using the bar to support division problems.

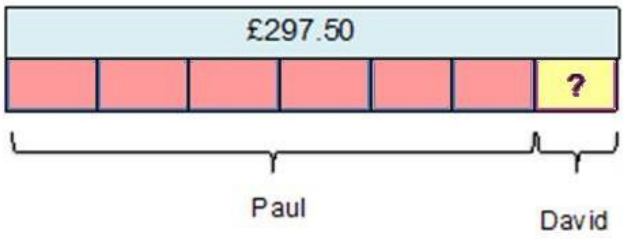
It is important for children to use the bar model in this way to encourage the use of it to aid with problem solving.</td><td>The total mass of 4 skips of rubble is 4920kg.
What is the mass of each skip?

  </td></tr>
<tr><td colspan="2"><u>Formal Written Expectation:</u>

Divide numbers up to four digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context</td></tr>
<tr><td colspan="2"><u>Key Vocabulary:</u>

Halve share; share equally; one each, two each, three each…; group in pairs, threes… tens; equal groups of; divide; division; divided by; divided into; remainder; factor; quotient; divisible by; inverse</td></tr>
</table>

<table>
<tr><th colspan="2">Year 6 Division</th></tr>
<tr><td>Divide at least four digits by both single-digit and two-digit numbers (including decimal numbers and quantities)</td><td>Short division with remainders: Pupils should continue to use this method, but with numbers to at least 4 digits, and understand how to express remainders as fractions, decimals, whole number remainders, or rounded numbers. Real life problem solving contexts need to be the starting point, where pupils have to consider the most appropriate way to express the remainder.
  </td></tr>
<tr><td>Divide using the long division method.</td><td>432 ÷ 15 becomes
  </td></tr>
<tr><td>Use the bar model to help divide.

It is important for children to use the bar in this way to encourage the use of it to facilitate problem solving.</td><td>Paul and David hire a car together at the cost of £297.50. Paul pays 6 times more than David. How much does David pay?
 
</td></tr>
<tr><td colspan="2"><u>Formal Written Expectation:</u>
Divide numbers up to 4-digits by a two-digit whole number using the formal written method of short division where appropriate for the context</td></tr>
<tr><td colspan="2"><u>Key Vocabulary:</u>
Share; share equally; one each, two each, three each…; group in pairs, threes… tens equal groups of; divide; division; divided by; divided into; remainder; factor; quotient; divisible by; inverse</td></tr>
</table>

Glossary of Mathematical Terms

2-digit number – a number with 2 digits like 23, 45, 12 or 60.

3-digit number – a number with 3 digits like 123, 542, 903 or 561.

Abstract – the "symbolic" stage of the CPA model, where children are able to use abstract symbols to model problems (Hauser). Once a child has demonstrated that they have a solid understanding of the "concrete" and "pictorial" representations of a problem, the teacher can introduce the more "abstract" concept using mathematical symbols (for example +, –, x, / to indicate addition, multiplication, or division).

Addition facts – knowing that 1+1 = 2 and 1+3 = 4 and 2+5 = 7. Normally we only talk about number facts with totals of 20 and under.

Array - an arrangement of a set of numbers or objects in rows and columns –it is mostly used to show how you can group objects for repeated addition or subtraction.

Bar model - is pictorial and it develops from children handling actual objects, to drawing pictures and then drawing boxes (bars) to represent objects. Eventually, they will no longer need to draw all of the boxes, which represent individual units, and instead they can just draw one long bar and label it with a number. The particular power of the bar modelling pictorial approach is that it is applicable across a large number of topics. Once students have the basics of the approach secured, they can easily extend it across many topics. A good understanding of the four operations and their associated calculation procedures (mental and written) is needed to use bar models. Children need to have efficient strategies to add, subtract multiply and divide for them to use bar models, as bar models don't provide the solution to a calculation answer – it only helps to **give an understanding of what to do to get to the answer**.

Bead String/Bar – a string with (usually 100) beads on, grouped by colour in tens. The bead string is a good bridge between a number track and a number line as it maintains the cardinality of the numbers whilst beginning to develop the concepts of counting 'spaces' rather than objects.

Bridging – when a calculation causes you to cross a 'ten boundary' or a 'hundred boundary' e.g. 85 + 18 will bridge 100.

Column addition – the name of the recommended written method for addition whereby the numbers are added in columns, 1s first then 10s and so on. Where the total exceeds 10, the ten 1s are exchanged for a 10 and written below the answer line. Sometimes referred to as 'carrying'.

Column subtraction – the name of the recommended written method for subtraction whereby the smaller number is subtracted from the larger, 1s first then 10s and so on. Where the subtraction cannot be completed as the second number is larger than the first, a 10 is exchanged (or renamed) for ten 1s to facilitate this.

Concrete - the "doing" stage of the CPA model, using concrete objects to model problems which brings concepts to life by allowing children to experience and handle physical objects themselves. Every new abstract concept is learned first with a "concrete" or physical experience. For example, if a problem is about adding up four baskets of fruit, the pupils might first handle actual fruit before progressing to handling counters or cubes which are used to represent the fruit.

Concrete apparatus – objects to help children count and calculate– these are most often cubes (multilink) but can be anything they can hold and move including the following: the actual physical items involved (e.g. lollipops); Cuisenaire rods; Dienes rods (hundreds, tens and units blocks); straws; Numicon; Place Value counters; place value arrow cards; number lines; and many more.

Count all – when you add by counting all the items/objects e.g. to add 11 and 5 you would count out 11, then count out 5, then put them together and count them all to get **16**.

Count on – when you add (or sometimes subtract) by counting onwards from a given number. E.g. to add 11 and 5 you would count on 5 from 11 (i.e. 12, 13, 14, 15, 16).

Decimal number – a number with a decimal point (e.g. 2.34, which would be said as two point three four).

Dienes Rods (or Base 10) – this is a set of practical equipment that represents the numbers to help children with place value and calculation. The Dienes rods show 1s, 10s, 100s and 1000s as blocks of cubes that children can then combine. Dienes rods do not break up so the child has to 'exchange' and "rename" them for smaller or larger blocks where necessary (e.g. One 10 equals 10 ones).

Difference – the gap between numbers that is found by subtraction (e.g. 7 – 5 can be read as "7 take away 5", or as "the difference between 7 and 5").

Dividend – the number being divided in a calculation.

Divisor – the smaller number in a division calculation.

Double – multiply a number by 2.

Efficient Methods – the method(s) that will solve the calculation most rapidly and easily.

Equals - is worth the same as (be careful not to emphasise the use of = to just show the answer).

Exchanging – Swapping a '10' for ten '1s' or a '100' for ten '10s' or vice versa (used in addition and subtraction when 'moving' 'ten' or a 'hundred' from its column into the next column and splitting it up). Heavily relied upon for addition and subtraction of larger numbers. Skills in this can be built up practically with objects, then Dienes rods/base 10, then place value counters before relying on a solely written method.

Expanded Multiplication – a method for multiplication where each stage is written down and then added up at the end in a column

Factor – a number that divides exactly into another number, without remainder.

Fluency – the ability to perform mathematical problems accurately and quickly. It one of the three core aims of the National Curriculum for maths, which aims to ensure that all pupils "***become fluent in the fundamentals of mathematics, including through varied and frequent practice with increasingly complex problems over time, so that pupils develop conceptual understanding and the ability to recall and apply knowledge rapidly and accurately"***.

Grid method – a method for multiplying two numbers together involving partitioning and multiplying each piece separately.

Grouping – an approach to division where the dividend is split into groups of the size of the divisor and the number of groups created are then counted.

Half - a number, shape or quantity divided into 2 equal parts.

Halve – divide a number by 2.

Integer - a whole number (i.e. one with no decimal point).

Inverse – the opposite operation. For example, addition is the inverse of subtraction and multiplication is the inverse of division.

Known Multiplication Facts – times tables and other number facts that can be recalled quickly to support with larger or related calculations e.g. if you know 4x7 then you also know 40 x 70, 4 x 0.7 etc.

Long Division – formal written of division where the remainders are calculated in writing each time (extended version of short division).

Long Multiplication – formal written method of column multiplication.

Multiple - a number which is an exact product of another number i.e. a number which is in the times table of another number.

Number bonds – 2 numbers that add together to make a given total, e.g. 8 and 2 bond to 10 or 73 and 27 bond to 100.

Number line – a line either with numbers or without (a blank numberline).

- The number line emphasises the continuous nature of numbers and the existence of 'in-between' numbers that are not whole. It is based around the gaps between numbers.
- Children use this tool to help them count on or count back for addition of subtraction. As they get older, children will count in 'jumps' on a number line e.g. to add 142 to a number they may 'jump' 100 and then 40 and then 2. The number line is sometimes used in multiplication and division but can be time consuming.

Number track – a sequence of numbers, each inside its own square. It is a simplified version of the number line that emphasises the whole numbers.

Numicon – practical maths equipment that teaches children the names and values of numbers 1-10 initially but them helps them with early addition, subtraction, multiplication and division. Numicon is useful for showing the real value of a number practically.

One-step calculation – a calculation involving only one operation e.g. addition. Usually the child must decide what that operation is.

Part, part, whole – involves children in seeing and experiencing numbers being made of two or more parts.

Partition – split up a larger number into parts, such as the hundreds, tens and units e.g. 342 can be partitioned into 300 and 40 and 2. (A very efficient technique for doubling and halving numbers!)

Pictorial - the "seeing" stage of the CPA model, using representations of the objects to model problems. This stage encourages children to make a mental connection between the physical object and abstract levels of understanding by drawing or looking at pictures, circles, diagrams or models which represent the objects in the problem. Building or drawing a model makes it easier for children to grasp concepts they traditionally find more difficult, such as fractions, as it helps them visualise the problem and make it more accessible.

Place Value – the value of a digit created by its position in a number e.g. 3 represents thirty in 234 but three thousand in 3567.

Problem solving - a fundamental means of developing mathematical knowledge at any level. Problem solving gives students a context to help them make sense out of the mathematics they are learning. Problems could be used to introduce new concepts or extend previously learned knowledge. It one of the three core aims of the National Curriculum for maths, which aims to ensure that all pupils ***"can solve problems by applying their mathematics to a variety of routine and non-routine problems with increasing sophistication, including breaking down problems into a series of simpler steps and persevering in seeking solutions***".

Remainder – a whole number left over after a division calculation.

Reasoning - the critical skill which enables us to make use of all other mathematical skills. With the development of mathematical reasoning, we recognise that mathematics makes sense and can be understood. It one of the three core aims of the National Curriculum for maths, which aims to ensure that all pupils
"reason mathematically by following a line of enquiry, conjecturing relationships and generalisations, and developing an argument, justification or proof using mathematical language".

Repeated addition – repeatedly adding groups of the same size for multiplication

Scaling – an approach to multiplication whereby the number is 'scaled up' by a factor of the multiplier (e.g. 4 x 3 means 4 scaled up by a factor of 3).

Sharing – an approach to division whereby the dividend is shared out into a given number of groups (like dealing cards).

Short Division - traditional method for division with a single digit divisor (this is a compact version of long division, sometimes called "the bus stop method").

Significant digit – the digit in a number with the largest value e.g. in 34 the most significant digit is the 3, as it has a value of '30' and the '4' only has a value of '4'.

Single digit – a number with only one digit. These are always less than 10.

Sum – the total of two or more numbers (it implies addition). Sum should ***not*** be used as a synonym for a calculation.

Two-step calculation - a calculation where two different operations must be applied e.g. to find change in a shop you will usually have to add the individual prices and then subtract from the total amount. Usually the child has to decide what these two operations are and the order in which they should be applied.